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An Integrated Math and Self-Regulated Learner Framework to Support Students with Learning

Disabilities and Math Anxiety

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Abstract

Students with math learning disabilities have been shown to experience math anxiety at rates nearly double those of students without. Anxiety about math is thought to disrupt learning by coopting attentional resources that could go towards problem solving and may be caused by the way in which students interpret their math-related experiences. This article describes a math intervention designed through a framework of self-regulated learning that defines self-regulated learners as students who are connected, self-aware, self-determined, strategic and resilient. Specifically described in this article is an intervention that helps students regulate anxiety, initiate a problem-solving strategy, and advocate as needed to use approaches they find effective.

Keywords: math anxiety, math learning disability, self-regulation, emotion regulation, cognitive flexibility

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Disabilities and Math Anxiety

Math anxiety is a state of discomfort caused by performing math-related tasks (Ma & Xu, 2004), and has been associated with physiological outcomes such as increased heart rates, neural activations similar to those in individuals experiencing pain (Lyons & Beilock, 2012), and high levels of extreme stress (Pizzi & Kraemer, 2017). Even though students with math-related learning disabilities have been found to experience math anxiety at nearly twice the rate as that of their typically developing peers (Devine, Hill, Carey & Szucs, 2018), math anxiety has also been reported to be prevalent at high rates among students with *high* levels of math achievement (Devine et al., 2018). Although these findings make the relationship between math anxiety and achievement challenging to understand, evidence supports the theory that anxiety triggers negative thoughts and emotions that co-opt the working memory needed to solve math problems (Ashcraft & Kirk, 2001). Put simply, math anxiety contributes to poor math performance.

At the same time, self-regulation deficits have been associated with both math anxiety (Jain & Dowson, 2009; Kramarski, Weisse & Kololshi-Minsker, 2010), and lower math achievement (Ramirez, Shaw & Maloney, 2018). Self-regulation in these studies has been defined as the coordination of cognitive and metacognitive processes to select and use appropriate strategies to achieve learning goals (Duncan & McKeachie, 2005; Jain & Dowson, 2009). In these cases, a lack of strategies to support self-regulated learning in math is thought to contribute to math anxiety – if a student does not have the tools to approach a math task, stress levels spike, which further compromises their ability to perform math tasks.

Finally, it has also been suggested that math anxiety is caused by a student's negative appraisal of their math learning experiences (Ramirez et al., 2018). When students evaluate their

math learning experiences as negative, they may begin to adopt a "failure as debilitating" mindset (Ramirez et al., 2018) that prevents them persisting with their math learning. This thought process can create a negative cycle of failure – stress – failure, that can cause a student to become highly anxious about math, and can significantly compromise their math performance.

Taken together, these findings suggest that intervention approaches to support students with math anxiety and low math performance should address these three underlying challenges – 1) math anxiety depletes cognitive resources, 2) limited strategic knowledge and flexibility contribute to math anxiety, and 3) students' negative perceptions and mind-sets contribute to math anxiety. In other words, students should be taught and encouraged to use techniques to manage their anxiety, to positively appraise their math learning experiences, and to develop their strategic thinking about approaching and solving math problems. This article describes the components of a multi-dimensional math and self-regulated intervention developed to address these three issues. The intervention described is grounded in a self-regulated learner framework and a math intervention framework developed by Johnson, Clohessy, and Chakravarthy (2018) to support students with math anxiety and math learning disabilities in 1:1 intervention setting.

Overview of the Self-Regulated Learner and Math Intervention Frameworks

Since different students will present with slightly different needs, an understanding of these frameworks can give practitioners the tools to tailor specific components as needed to respond to the unique needs of their learners. For example, the student for whom the intervention detailed in this article was designed had very high levels of math anxiety that often prevented her from getting started with a math task. She also had very compromised working-memory ability, low competence with self-regulated learning strategies, low confidence levels and was struggling with long division in her fifth-grade math class. A different student may present with a different

variation of conditions. Through the use of the two frameworks to guide intervention design, the specific strategies can be adapted to support a student's unique needs. For example, the math strategy will vary depending on the math concept to be taught, or the technique to support math anxiety may differ depending on how anxiety manifests for the student.

Self-Regulated Learner Framework

Self-regulated learning is the ability to regulate one's thinking, behavior, and emotions in pursuit of a learning goal. Self-regulation has been strongly linked to successful learning and educational outcomes and it is critical for success in school. (Durlak, Weissberg, Dymnicki, Taylor, & Schellinger, 2011; Zimmerman & Schunk, 2008). However, self-regulation is a multifaceted, dynamic construct. This means that students may be able to demonstrate some aspects of self-regulation, but not others; it also means that students may be able to self-regulate in some contexts but not others. To reflect this complexity, we define self-regulated learners as students who are connected, self-aware, self-determined, strategic and resilient (Johnson, Clohessy, Meek, & Spears, 2018). We define each of these five components of self-regulation below.

Connected learners feel safe and as if they belong. They are supported to be themselves and are socially aware; they can take the perspective of others; and they are able to develop, manage, and maintain healthy relationships. Students who trust and feel connected to their teachers feel safe and tend to be more engaged during the learning process, which can lead to higher academic achievement. Students' intrinsic motivation and mind-set are enhanced through positive feedback from teachers and through student-centered teaching environments (Leighton & Bustos Gomez, 2018). Self-aware students understand their strengths and needs as learners, they understand how their actions affect themselves and those around them, and they understand which learning approaches and habits are most effective for them. Self-determined learners are

self-directed and have authentic agency over their learning. They are able to set goals, make plans, and monitor their progress to reach their goals. *Strategic* learners have a large repertoire of learning strategies and are able to select and effectively use the appropriate strategy to reach their learning goal (Karlen, 2016). Cognitive strategies are required to process, transform, and organize information. Metacognitive strategies are taught to support the planning, monitoring and evaluation of learning. Students are also taught environmental strategies to create the conditions and routines that effectively support learning (Ben-Eliyahu & Linnenbrink-Garcia, 2015). Finally, *resilient* learners are able to use strategies to recover from difficulty or disappointment and are able to adapt to stress or adversity. Cognitive flexibility, the ability to approach a problem in a different way, and emotion regulation are important in developing resilience (Zolkoski & Bullock, 2012). Figure 1 depicts the self-regulated learner framework used to inform intervention planning.

Math Intervention Framework

As depicted in Figure 2, the math intervention framework includes teaching for conceptual and procedural understanding, developing a student's mathematical reasoning ability and their ability to make connections across math concepts. The intersection of these aspects of math is strategic competence with problem solving, demonstrated when a student can independently identify and apply an effective and efficient approach to solving a given mathematics problem. Conceptual understanding is defined as the understanding of the principles that govern a math domain (Rittle-Johnson & Schneider, 2015). Procedural understanding is the knowledge of action sequences for solving problems (Rittle-Johnson & Schneider, 2015).

Students with greater conceptual understanding of math concepts tend to develop greater procedural skill, and it has also been shown that students who develop strong procedural

understanding can improve their conceptual understanding (Rittle-Johnson & Schneider, 2015). Mathematical reasoning is the ability to construct mathematical conjectures, to develop and evaluate math arguments and to select and use various types of mathematical representations (NCTM, 2000). Math connection is the ability to see the specific relationships between mathematical concepts (NCTM, 2000).

The math instructional framework is not meant to depict a sequential instructional preference, as research suggests that students develop skills across these aspects iteratively (Rittle-Johnson & Schneider, 2015). Rather, the goal is to ensure that intervention is designed to support students' math understanding through these four areas. For nearly two decades there have been calls for all students to receive math instruction that leads to strong conceptual understanding, procedural fluency, strategic competence in problem solving, mathematical reasoning and productive dispositions towards mathematics (National Research Council, 2001). An extensive research base identifies a number of instructional practices that are effective for SWD to develop strong competency in math, including: 1) explicit instruction of the mathematical concept and procedures to support student understanding (Doabler, Baker, Kosty, Smokowski, Clarke, Miller & Fien, 2015), 2) visual representations to support students' conceptual understanding and their ability to connect the math concept to the math procedures (Gersten, Chard, Jayanthi, Baker, Morphy & Flojo, 2009), 3) cognitive strategy instruction that teaches students to identify and solve a variety of problem types (Griffin & Jitendra, 2009), 4) explicit inquiry routines to support students' strategic competence and ability to solve complex equations (Impecoven-Lind & Foegen, 2010; Scheuermann, Deshler & Schumaker, 2009), and 5) teaching students to verbalize their math reasoning and using mathematical vocabulary

(Gersten et al., 2009). These evidence-based practices form the basis for the instructional approaches used in the integrated self-regulation and math intervention.

An Integrated Intervention to Reduce Anxiety and Improve Math Performance

In this section, the application of the two frameworks to create a specific intervention designed to: a) reduce math anxiety to avoid depletion of cognitive resources and to build student confidence, b) teach a strategy that promotes flexibility in solving problems, and c) support student's ability to better appraise stressful situations, and to adopt a 'failure as enhancing learning' mindset, is presented. The intervention was designed for use within an individualized intervention scenario, but it can be adapted for use with small groups. Figure 3 includes a completed intervention template. Each part of the template is described below to facilitate application to students with different presenting concerns.

Step One: Identify Students' Specific Needs

The first part of intervention design involves identifying the student's specific concerns. First, it is important to note how and when a student's math anxiety manifests. For some students, timed assessments may be extremely anxiety inducing, for others, fear of making an error may be preventing them from moving forward, and for still others, extreme anxiety that prevents them from getting started may be the issue. Next, if the student has a completed psychoeducational evaluation, it is important to note what specific cognitive processes are compromised for this student. Having this information readily available can help the intervention teacher have greater levels of empathy, and help inform accommodations to support learning. In the case presented here, the student had significant challenges in working memory and receptive language. Then, identify the math concept of focus, in this case long division. Finally, observe and note the student's response to set-backs. Do they present with a "failure-as-debilitating"

mindset, indicated by shutting down if they get something wrong, avoiding math, or saying things like "I'll never be good at math"? Or do they present with a "failure-as-enhancing" mindset, indicated by a willingness to review and learn from errors?

Step Two: Strategy to Reduce Anxiety

Math related anxiety and stress have been shown to compromise working memory and attentional resources (Ashcraft & Kirk, 2001). Therefore, it is critical to include a strategy or technique to help a student lower their anxiety levels. Developing a safe and positive learning environment through positive feedback should always be a priority in the math intervention setting (Leighton & Bustos Gomez, 2018). Establishing trust and a strong connection with students can help to lower anxiety. Some examples of positive feedback are included on the intervention plan to remind the intervention teacher to pay attention to the way in which she responds to the student to create a positive and safe environment.

In addition, for students who experience significant anxiety, a targeted technique may be needed. As depicted in Figure 3, for this student, intervention begins with an opening routine that the student can confidently and independently do. This student's intervention began with a "10 cloud" activity, in which the student would draw a cloud on a white board, and then write out all of the ways to construct the number 10 (e.g. 3 + 7 = 10; 2 + 8 = 10). This routine was chosen for this student because she had a strong knowledge of addends to 10 and she enjoyed the opportunity to write on the board in the 1:1 setting since she typically avoided volunteering to do so in front of her classmates in the general education classroom. The goal of this intervention step is two-fold. First, it serves as a reminder to this student that she has competency in math. Second, it served as a calming routine, which helped her get started with the lesson, to orient her mind towards math, and to have an early success in the intervention period.

Step Three: Math Strategy to Build Conceptual, Procedural, Reasoning and Connections

The next step focuses on teaching a strategy or strategies to address the particular math domain of focus. In this particular case, the student was struggling with long division. In her general math class, students were taught to use a partial quotient model, but this student did not feel confident in her ability to use the strategy, and this would trigger a negative cycle of making lots of errors, which further convinced her that she could not complete the task. It is highly likely that here working memory challenges made the use of the partial quotient model quite difficult. Drawing on evidence-based practices for math instruction, the student was taught a strategy to use a Hundreds, Tens, Ones (HTO) Chart, and the Concrete – Representational – Abstract (CRA, Steedly, Dragoo, Arafeh & Luke, 2008) progression. The steps for this strategy are outlined in Figure 4. The strategy was first taught using base-10 blocks and the HTO chart, which the student had used to develop her knowledge of place value and ability to add and subtract with regrouping. Instruction progressed through the representational and then abstract, and the student was explicitly shown the connections across the three. This approach supported the student's conceptual understanding and connected that conceptual understanding to the underlying procedures. The student was supported to make connections across math concepts (i.e. in this case how to regroup using the HTO chart, using base-10 blocks to build numbers) using tools and supports that were familiar to her because she used them to learn other math concepts. Throughout the intervention, the student was encouraged to explain her reasoning as she worked through the various steps.

Step Four: If-Then Strategy to Develop Flexible Thinking and Positive Mindset

The final part of this intervention was designed to help this student become more flexible and positive in her response to errors. When intervention first began, the student would become

very distraught when she made an error, sometimes shutting down for the remaining intervention time. To build her resilience and give her a strategy to continue working, an "if-then" approach was used. "If-then" planning is used extensively in behavioral change to provide a plan for managing obstacles encountered in pursuit of a goal (Baumeister, Vohs & Tice, 2007). The plan is developed and rehearsed before challenges are encountered.

For this student, the "if-then" plan was, "If I make an error in solving a problem, I will use the *Break it Down* strategy to review the steps, and my reasoning." The *Break it Down* strategy is a graphic organizer (see Figure 5) to lead the student through the process. This strategy accomplishes two things. First, creating an "if-then" plan signals to the student that making errors is an expected part of learning math. Second, the "if-then" cue card is always placed on the student's desk as a prompt for what to do next, and provides the student to use the tools they have learned to solve problems. This process should be scaffolded by the intervention teacher, with the goal of supporting the student to become independent over time.

Step Five: Evaluating the Strategy

Both informal and formal measures can be used to assess the effectiveness of the strategy. As is the case in any intervention setting, the routine evaluation of data through a databased decision-making process is critical for successful outcomes.

Assessing Math Anxiety. The Math Anxiety Scale for Children (MASC; Chiu & Henry, 1990) can be used as a pre and post intervention measure to first identify the level of a student's anxiety, and then to have an indication of whether anxiety has been decreased over time. The MASC is designed for students in grades 4 through 8, but there are other measures of math anxiety for various age ranges. Because students will experience anxiety in different ways it is important to use observation and a relevant metric for measuring change over time. For example,

in the present case, initially, the student took a long time to get started with a math problem. The intervention teacher created a graph to set goals with the student to reduce the time to get started. This promotes both *self-awareness* in the student (i.e. helping the student recognize it will be challenging to develop math skills if they do not use intervention time), and *self-determination* (i.e. setting and monitoring progress towards goals can be a strong motivator for students).

Assessing Math Performance. Curriculum-based measures are used as a general outcome measure for students receiving intervention. In addition, a mastery measure based on number of trials correct under scaffolded and non-scaffolded conditions gives important instructional feedback to the intervention teacher. Students with significant learning challenges need sufficient opportunities to achieve mastery.

Conclusion

Students with math related learning disabilities are more likely to experience math anxiety than their typically performing peers (Devine et al., 2018). Currently, the interventions used with SWD typically target lower level skills such as fact recall or competency with procedures only (Marita & Hord, 2017), and most do not address self-regulation or math anxiety directly. The approach to intervention as described in this article presents a promising way to address the comprehensive needs of students with significant learning challenges, anxiety and self-regulation deficits. The success of this intervention design will depend on teachers' understanding of the self-regulation and math intervention frameworks. More information about both is available on the website http://www.lplearningcenter.org. Our comprehensive model addresses the various aspects of anxiety, self-regulation, and math strategy knowledge to help students learn to persist in math and understand that productively struggling in math is a natural part of learning (Hiebert & Grouws, 2007).

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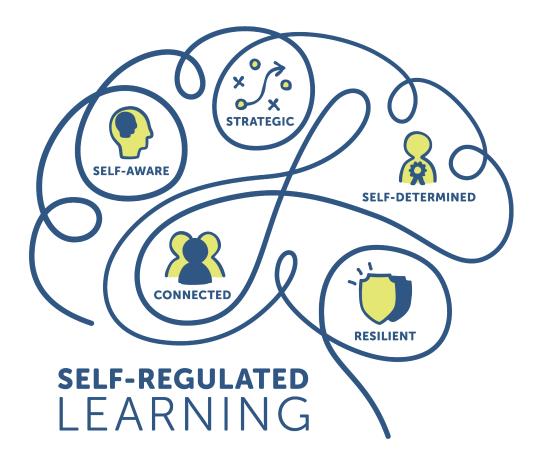


Figure 1. Self-Regulated Learner Framework

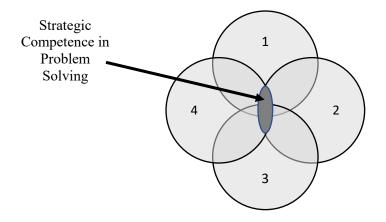


Figure 2. Math Instructional Framework

- Conceptual Understanding
 Procedural Understanding
 Math Reasoning

- 4. Math Connection

Self-Regulation and Math Intervention Template

1. Identify Students' Specific Needs

| Anxiety | Cognitive Process | Strategy | Mind-Set |
|------------------------|---------------------------------|---------------------|--------------------------|
| Susie's* anxiety level | Significant working | Use an HTO chart | Susie shuts-down |
| is high, she has | memory limitations | and CRA progression | when she makes an |
| difficulty getting | (< 10 th percentile) | for long-division | error, it reinforces her |
| started with math | | _ | feeling that she |
| intervention | | | cannot do math |

2. Strategy to Reduce Anxiety

| Positive Feedback from Teacher (example) | Calming Strategy |
|---|--|
| "Let's check your work together" | Begin each session with "10 cloud" strategy, |
| "Can you talk me through the steps you | prompt to use this (on board or on paper) |
| took?" | when anxiety is heightened; work towards |
| "You worked really hard to solve that | student independent use of strategy (e.g. no |
| problem." | prompting from teacher) |
| "It's good to learn that that approach didn't | |
| work, let's think about how else we can | |
| approach the problem." | |

3. Math Strategy to be taught

| Long Division Problem Solving Template (CRA Progression) | | | | |
|--|---------------|---|--|--|
| Write the Division Problem: $142 \div 3$ | | | | |
| Hundreds | Tens | Ones | | |
| | | 日日 | | |
| Make groups of 3 (break the hundred to 10 tens) | | 000 000 000 000 000 000 000 | | |
| 0 groups of 3 | 4 groups of 3 | 7 groups of 3 R 1 | | |

Answer: $142 \div 3 = 47 R 1$

4. If-Then Planning

| If | Then |
|---|--|
| If my answer isn't correct the first time | Then I can use my Break it Down sheet to |
| | redo the problem and check my work, and if I |
| | need to, I can do my 10 cloud first. |

5. Evaluation

- 1. Pre/post Math Anxiety Scale for Children
- 2. Observation data how long does it take to get started with math lesson, how much time on task during the lesson?
- 3. CBM data (bi-weekly progress monitoring)
- 4. Mastery data student can use the long division problem solving approach to consistently and accurately solve problems without teacher prompt. Support student to move from concrete to representational.

Strategy for Long Division

Materials: Base-10 blocks, Marker, Place Value chart (HTO chart)

Objective: The objective is to divide 142 cubes in groups of 3

Divide $142 \div 3$

Method: Using Concrete

- Step 1: Place 1 hundred (flat) in the 100's place, 4 tens in the 10's place and 2 ones in the 1's place.
- Step 2:. Start at the 100's place; since we cannot make a group of 3, we decompose 100 into 10 tens (10 longs).
- Step 3: Now there are, 10 + 4 = 14 tens. We can make 4 groups of 3 and have 2 tens remaining.
- Step 4: Decompose the 2 tens into 20 ones.
- Step 5: Now there are, 20 + 2 ones = 22 ones. We can make 7 groups of 3 and have 1 one remaining.
- Step 6: Write down the number of groups formed under each place value position.
- Step 7: Write down the answer: $142 \div 3 = 47 R 1$

Figure 4. Instructions for using the HTO chart

Break It Down

| Word problem | |
|---|--------------------|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| What do you know? | |
| | |
| | |
| | |
| What do you need to find? | Solve the problem. |
| | |
| | |
| | |
| | |
| What operation (s) will you use to solve the problem? | Check your answer. |
| | |
| | |
| | |